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ON TWO THEOREMS OF QUINZII AND RENT CONTROLLED HOUSING ALLOCATION IN SWEDEN
or
"I buy your slithy tove only if my best friend's wife buys your best friend's husband's slithy tove"

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The Swedish rent control system creates a white market for swapping rental contracts and a black market for selling rental contracts. Empirical data suggests that in this black-and-white market some people act according to utility functions that are both discontinuous and locally decreasing in money. We discuss Quinzii's theorem for the nonemptiness of the core of generalized house-swapping games, and show how it can be extended to cover the Swedish game.

In a second part, we show how this theorem of Quinzii and her second theorem on nonemptiness of the core in two-sided models are both special cases of a more general theorem.

Keywords: Core; house-swapping; rent control; indivisible goods.

Subject Classification: 91A06, 91B68

The inspiration to this paper comes from two very different sources: a couple of game-theoretic theorems of Quinzii on the one hand, and the Swedish rent controlled market for housing on the other hand.

To begin with the theoretic side, Quinzii (1984) introduced a very general model for trade of indivisible goods, such as houses. Quinzii's paper has two parts, with the first part treating the house-swapping type of markets and the second part in a similar way dealing with two-sided matching markets. The main result of each part is a theorem stating that the core is nonempty under certain conditions.

Of course, housing allocation is studied not only in game theory but also in other branches of economics. For instance, recent empirical work studies the effects of rent control on the allocation of housing in some American cities [Glaeser and Luttmer (2003), Glaeser (2003)]. In the first section of the present paper we will discuss rent control in Sweden, and how it results in utility functions that are not allowed in Quinzii's game-theoretic model of housing allocation. We will then show that Quinzii's first theorem can be generalized to cover also the Swedish market.

In the second section we will discuss the second theorem of Quinzii, which deals with two-sided matching. We will develop a more general framework of *circular exchange economies* for which we prove a theorem of core nonemptiness which contains both Quinzii's theorems as special cases. We also discuss the relation of our result to the even more general framework of Quint (1997).

1. Quinzii's First Theorem and Swedish Rent Controlled Housing

In this section we first describe the Swedish system for rent control and how it seems to result in exotic utility functions, with utility as a function of money making discontinuous and decreasing jumps. The first theorem of Qunizii (1984) does not apply to such utility functions, but we show how it can be generalized to show that the core of the Swedish game is nonempty.

1.1. *Rent control in Sweden*

Under the Swedish system for rent control on housing, rents should be at a level determined by the principle of *"bruksvärde"* (literally: value-of-usage). In practice, private landlords must not set rents significantly higher than for similar apartments owned by a municipal housing company. The municipal housing companies set their rents after negotiations with the local tenants' union. By tradition, the intra-city variation in demand of housing is not taken into account in the rent level, although there are no regulations against the parties agreeing to do so. For example, to most people an apartment in downtown Stockholm is much more attractive than an apartment of identical size and standard in a worn-down and distant suburb. Since the rent is the same, the demand on rental contracts downtown is so high that there are almost never any vacancies. Most people who move from such an apartment will not simply leave such an attractive contract, but often try to sublet the apartment. Another option is to use the contract as part of a deal, either by swapping it against a new contract, or by selling the contract.

Although it is legal for two tenants to swap contracts, it is explicitly illegal to sell rental contracts in Sweden. Therefore a black market has emerged where contracts are swapped with money exchanged under the table. Neither buyer nor seller is inclined to report the criminal deal, so it is not considered a high risk venture to engage in this black market. In fact, during an entire decade (1990–1999) only about a dozen people were found guilty of this crime [Tufvesson and Ljungkvist (2001)].

The disadvantages of the Swedish housing market regulation were pointed out already forty years ago [Bentzel, Lindbeck and Ståhl (1963)]. For a recent review, see the volume on rent control edited by Ellingsen and Englund (2003). The rent control system has enjoyed wide support from the Swedish public and politicians. Ellingsen (2003) gives a median-voter type explanation of this political support. However, in the last decade there have been signs that a shift in the political climate might be under way. Most notably, the municipal housing company and tenants' union of Malmö (the third largest city in Sweden) agreed in the early nineties to move toward market oriented rents. The effects of this Swedish experiment of market rents, in comparison to the development in Stockholm where no movement toward market rents have occurred, were recently studied by Lind and Hellström (2003). They found that economic segregation (higher incomes in more attractive areas) has increased just as much in Stockholm as in Malmö during this decade. As a partial explanation, Lind and Hellström points to the Stockholm centered boom in the IT sector in the late nineties, when people with suddenly much higher incomes demanded housing in downtown Stockholm. When households already living in this area saw that the gap between their rent and the market rent widened, it opened a profitable possibility of selling the rental contract on the black market — where only people with higher incomes can afford to buy.

In 2001, the association of landlords in Stockholm made a survey to their members about swapping of rental contracts [Tufvesson and Ljungkvist (2001)]. Although the landlords cannot know for sure which swaps include money under the table, they see certain indications of the likeliness of a deal being black. The landlords estimate that 50 percent of all swaps were black-market.

Our conclusion from all this evidence is that both the black and white market of swapping rental contracts play important roles for adjusting allocation of rental housing in Sweden. Many people make deals on the black market, but just as many deals are white. So why are there any white deals at all? A recent questionnaire study by Eriksson and Lind (2004) reported the following findings among students at universities in Stockholm who were presented with a hypothetical case of black market trading:

30 percent would buy at the current market price. Another 42 percent would buy if they managed to obtain a bargain price. 28 percent would never buy on the black market.

49 percent would sell at the current black market price. Another 10 percent would sell if they managed to obtain an overprice. 41 percent would never sell on the black market.

It seems that we can roughly speak about three types of buyers, see Fig. 1. An agent of the first type finds buying on the black market unproblematic, so given a particular house she might buy we can assume her utility function to be continuous and increasing in money. An agent of the second type wants a risk premium for engaging in the black market, so her utility function will have a discontinuity at zero price. An agent of the third type completely avoids the black market, so her utility function will be $-\infty$ for prices above zero.

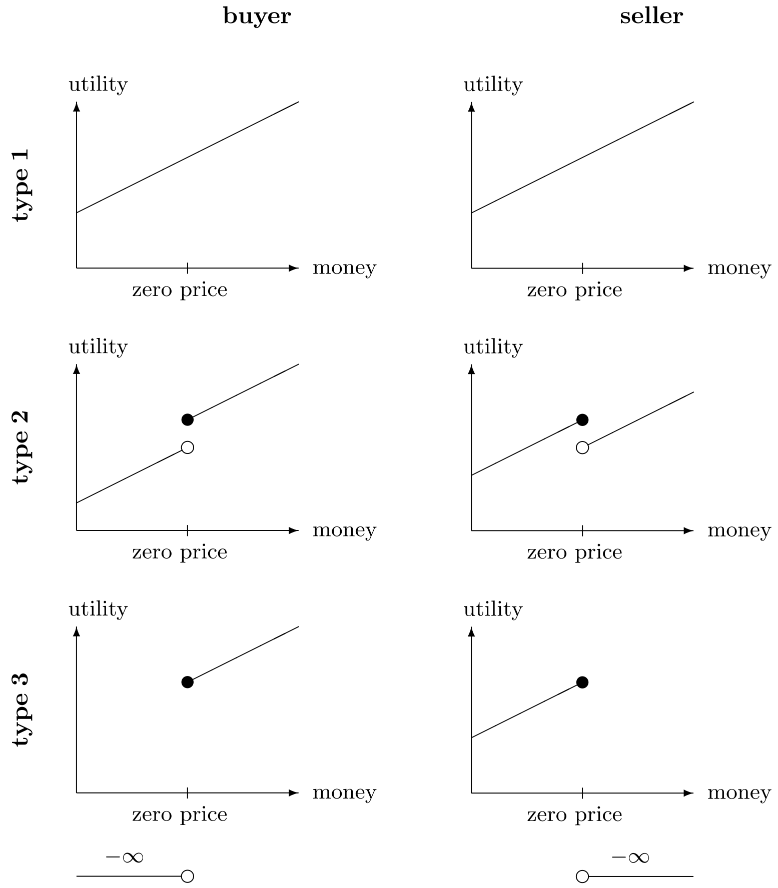

Fig. 1. The utility functions for the three types of buyers and sellers. On the x-axis is the amount of money the agent owns after she has bought or sold the house.

On the seller side, the risk premium demanded by the second type of agent now causes her utility function to be decreasing and discontinuous at zero price: obtaining a price slightly higher than zero is worse than a deal at zero price. For the third type of agent, all sales at a price greater than zero give her a utility of $-\infty$.

1.2. *The Swedish game and Quinzii's theorem*

The first game-theoretic model of a house-swapping market was presented by Shapley and Scarf (1974). This is a game where each player owns a house when entering the market, wishing to leave the market with a better house if possible. The values of houses are private, not common, so each player maintains her own preference order on all houses in the market. In this model houses are taken as prototypical representatives of indivisible goods; players cannot buy half a house.

The houses are the only goods available in this game, so there is no money around that could make a player change her preferences.

The house-swapping game of Shapley and Scarf models the white market of rental contracts in Sweden. In the black market, the participants make deals at a negotiated money exchange. When money exchange is possible we obtain the TU (transferable utility) house-swapping game, first considered by Tijs *et al.* (1984) under the name of "permutation game". In a seminal paper, Quinzii (1984) introduced a model that contains both the house-swapping game and the permutation game as special cases. In this general model, the *exchange economy*, each agent starts out with at most one house and an initial endowment of money. Every agent i has preferences represented by a utility function $u_i(m, h)$ where m is a quantity of money and h is some house (or no house). A feasible allocation for this economy is a distribution of the houses, with at most one house to each agent, together with a distribution of the sum of the initial endowments. (In Quinzii's formulation it is acceptable for money to disappear, since she has assumed her utility functions to be nondecreasing, but such free disposal of money is not reasonable if we allow decreasing utility functions.) Feasible allocations for smaller coalitions S are defined in the obvious way. The core consists of those allocations which are feasible and such that for no coalition S is there an allocation feasible for S which is strictly preferred by all its members.

Theorem 1 [Theorem 1 of Quinzii (1984)]. *If utility functions are continuous and non-decreasing in money, then the exchange economy has a nonempty core.*

Quinzii's nonconstructive proof is an application of the theorem due to Scarf (1967) which says that a balanced game has nonempty core. Both conditions on the utility functions are implicitly used by Quinzii. We have already mentioned the importance of nondecreasingness. Continuity is needed when Quinzii defines

$$m_{ih}(v) = \inf\{m_i \in \mathbb{R}_+ \,|\, u_i(m_i, h) \geq v_i\} \tag{1}$$

for a vector v of utility levels, and she demands that the infimum is actually attained if the set $\{m_i \in \mathbb{R}_+ \,|\, u_i(m_i, h) \geq v_i\}$ is nonempty.

Quinzii's theorem can now be extended.

Theorem 2. *Add $-\infty$ to the set of legal values of utility functions. Suppose that, for every agent i and every house h, the supremum*

$$u_i^*(m, h) = \sup\{u_i(x, h) \,|\, 0 \leq x \leq m\}$$

is attained for all $m \geq 0$ and is right-continuous as a function of m. Also, suppose that at least one agent has a utility function that is non-decreasing in money. Then the exchange economy has a nonempty core.

Proof. First, we let all agents pretend that their utility functions are u_i^* instead of u_i. By construction, u_i^* is non-decreasing in money, and since it is right-continuous, $\inf\{m_i \in \mathbb{R}_+ | u_i^*(m_i, h) \geq v_i\}$ is attained for any v_i if the set is nonempty. Thus, Quinzii's proof goes through as before when applied to the starred utility functions. Consider an allocation A in the core of this game, where agent number i has the money m_i and the house h_i (or no house).

Now, let the agents stop pretending and go back to their original utility functions. Since $u_i \leq u_i^*$, an agent i that was happy according to u_i^* may now be less happy because she has too much money. She would like to get rid of the money $m_i - m_{\sup}$, where $u_i(m_{\sup}, h_i) = \sup\{u_i(x, h_i) \mid 0 \leq x \leq m_i\}$. But since at least one agent has a utility function that is non-decreasing in money, the other agents can dispose of any unwanted money by giving it to her.

Thus, we have found an allocation B such that every agent is at least as happy according to u_i as she was in A according to u_i^*. Since $u_i^* \geq u_i$ this implies that B lies in the core of the game with the original utility functions. □

Remark. The importance of right-continuity can be demonstrated by the following example of an economy of just three agents and one house. Suppose agents 1 and 2, who both have an initial money endowment of 50, both want the only house h owned by agent 3. The utility that agents 1 and 2 derive from having this house together with money m is a left-continuous function:

$$u_1(m, h) = u_2(m, h) = \begin{cases} m + 100 & \text{if } m > 0, \\ 0 & \text{if } m \leq 0. \end{cases}$$

In words, they don't like being completely broke. On the other hand, if they do not get the house, these agents experience utilities

$$u_1(m, \emptyset) = u_2(m, \emptyset) = m.$$

Finally suppose that agent 3 has the simple utility functions $u_3(m, h) = u_3(m, \emptyset) = m$. The core of this economy is empty, since both agents 1 and 2 are willing to overbid the other until someone gets completely broke, but no one is willing to make that limiting bid that makes him completely broke.

Corollary 1. *In an exchange economy where all players have utility functions of one of the three Swedish types, and at least one seller is of the first type, the core is nonempty.*

Proof. Let u be a utility function of one of the types depicted in Figure 1. If u is non-decreasing in money then $u^* = u$. Otherwise, u is a sellers' function of type 2 or 3, and these have well-defined u^* which are continuous. u^* is non-continuous only if u is a buyers' function of type 2 or 3, and these are right-continuous, so Theorem 2 applies. □

2. Circular Exchange Economies and a Common Generalization of Quinzii's Two Theorems

Consider our usual exchange economy with the assumption that everyone has a house (this is no real restriction, since we can always introduce worthless houses.)

Quinzii's second theorem is about two-sided matching markets regarded as exchange economies where allocations must be made pairwise, so that the house of the ith agent ends up at the jth agent if and only if the converse also holds.

Theorem 3 [Theorem 2 of Quinzii (1984)]. *If utility functions are continuous and non-decreasing in money, and allocations must be made pairwise* (*where the two agents belong to different sides*), *then the exchange economy has a nonempty core.*

In general, forgetting about twosidedness, a feasible allocation (for any coalition) can be described by a permutation π of the members of the coalition, such that the ith agent gets her house from the $\pi(i)$th agent. We can introduce various restrictions on π, and see how this affects the core. Our aim in this section is to prove one general result that encompasses both Theorems 1 and 2 of Quinzii (1984) as natural special cases.

In a sense, such a general theorem has already been given by Quint (1997). He defined a notion of *strongly balanced* restrictions on permutations, and proved that the core is nonempty if and only if the game is strongly balanced. However, one important aspect of Quinzii's results is that the technical notion of balancedness does not enter the definition of the feasible allocations; on the contrary, her two examples have natural definitions, and she proves that each of them is balanced. Our aim is to use her technique and find the most general class of restrictions for which this technique proves balancedness. We call this class *circular exchange economies.* Thus, we aim for a special case of Quint's theorem that shares the directness of Quinzii's two theorems, and from which both of Quinzii's theorems can be derived directly without any need to prove balancedness.

We begin by defining Σ-restricted exchange economies and show when they are balanced. We do this following Quinzii (1984) as closely as possible, but alternatively the result can be derived from the work of Quint (1997). Then we define the original concept of circular exchange economy and give sufficient conditions for the core to be nonempty.

2.1. Σ-*restricted exchange economies*

Let N be the number of agents.

Definition 1. A family B of coalitions $S \subseteq [N] := \{1, \dots, N\}$ is *balanced* if there are positive rational numbers $(\delta^S)_{S\in B}$ such that, for each $i \in [N]$,

$$\sum_{i\in S\in B} \delta^S = 1.$$

Definition 2. A game is *balanced* if, for every balanced family B of coalitions, $v \in V([N])$ if $v \in \bigcap_{S\in B} V(S)$, where $V(S)$ is the set of realizable utility vectors for the coalition S.

The reason for defining balanced games is that we want to apply the useful theorem of Scarf (1967) which says that any balanced N person game has a nonempty core.

For a coalition $S \subseteq [N]$, an *S-permutation matrix* is an $N \times N$ zero-one matrix containing one 1 in each row and each column indexed by a member of S, and zeros in rows and columns indexed by $[N] \backslash S$. An S-permutation matrix σ^S can equivalently be regarded as a permutation of the set S, such that $\sigma^S(i) = j$ if and only if $\sigma^S_{i,j} = 1$.

By a *permutation matrix* we will mean an S-permutation matrix for any $S \subseteq [N]$. Let Σ be any set of permutation matrices. For a coalition S, the set of S-permutation matrices in Σ is denoted by Σ^S. Define $\langle \Sigma \rangle$ to be the vector space of matrices generated by Σ over the rationals:

$$\langle \Sigma \rangle := \left\{ \sum_{\sigma \in \Sigma} a_\sigma \sigma \;:\; a_\sigma \in \mathbb{Q} \right\}.$$

A matrix is *doubly stochastic* if all its entries are nonnegative real numbers, and if each of its rows and columns sums to 1.

Definition 3. An exchange economy is Σ-*restricted* if the permutations in Σ are the feasible allocations of houses.

Definition 4. Σ is *balanced* if every doubly stochastic matrix in $\langle \Sigma \rangle$ belongs to $\langle \Sigma^{[N]} \rangle$.

Theorem 4. *A Σ-restricted exchange economy is balanced if Σ is balanced.*

The proof is identical to the proof of Theorem 1 of Quinzii (1984), once we have established the following lemma.

Lemma 1. *If Σ is balanced, then the following holds. Let $M = (m_{i,j})$ be any $N \times N$-matrix with entries in $\mathbb{R} \cup \{+\infty\}$, and let B be any balanced family of coalitions $S \subseteq [N]$ with weights $(\delta^S)_{S \in B}$. Then,*

$$\min_{\sigma \in \Sigma^{[N]}} \sum_{i=1}^{N} m_{i,\sigma(i)} \leq \sum_{S \in B} \delta^S \min_{\sigma^S \in \Sigma^S} \sum_{i \in S} m_{i,\sigma^S(i)}. \tag{2}$$

Proof. Let LH and RH denote the left-hand side resp. right-hand side of (2). Let $(\sigma^S)_{S \in B}$ be minimizing arguments of RH. Observe that $\sum_{i \in S} m_{i,\sigma^S(i)} = \operatorname{tr} M\sigma^S$. This gives us

$$\mathrm{RH} = \sum_{S \in B} \delta^S \operatorname{tr} M\sigma^S = \operatorname{tr} \left(M \sum_{S \in B} \delta^S \sigma^S \right).$$

The matrix $\sum_{S \in B} \delta^S \sigma^S$ is doubly stochastic (since B is a balanced family) and lies in $\langle \Sigma \rangle$. Thus it also belongs to $\langle \Sigma^{[N]} \rangle$ so it can be written as $\sum_{\sigma \in \Sigma^{[N]}} \gamma_\sigma \sigma$ for some rational numbers γ_σ which sum to 1. Therefore we have

$$\mathrm{RH} = \operatorname{tr} \left(M \sum_{\sigma \in \Sigma^{[N]}} \gamma_\sigma \sigma \right) = \sum_{\sigma \in \Sigma^{[N]}} \gamma_\sigma \operatorname{tr} M\sigma \geq \sum_{\sigma \in \Sigma^{[N]}} \gamma_\sigma \,\mathrm{LH} = \mathrm{LH}.$$

□

Remark. Theorem 4 also follows from Theorems 4.1 and 4.2 in Quint (1997), since Σ is *strongly balanced* in the sense of Quint if it is balanced in our sense.

2.2. *Circular exchange economies*

Now, we study an exchange economy where the agents are divided into $k \geq 1$ groups $G_1, \ldots, G_k$ of size n. A feasible allocation is a distribution of the houses such that the agents in G_1 get their houses from the agents in G_2, which get their houses from G_3, and so on, until finally G_k get their houses from G_1. Let us call this a *k-circular* exchange economy.

Define the k-circular addition operator $\oplus$ by $i \oplus 1 = i + 1$ if $i < k$ and $k \oplus 1 = 1$. An $nk \times nk$ matrix A can be thought of as a $k \times k$ block matrix $A' = (A'_{i,j})_{i,j \in [k]}$ consisting of $n \times n$-blocks $A'_{i,j}$. We say that A is *k-circular* if $A'_{i,j} = \mathbf{0}$ unless $j = i \oplus 1$. Then a feasible allocation for a coalition in a k-circular exchange economy may be represented by a k-circular permutation matrix of size $nk \times nk$.

Let $\mathcal{M}_n$ be the linear vector space of all $n \times n$ rational matrices over the rationals. A linear map $f : \mathcal{M}_n \to \mathcal{M}_n$ is *permutation preserving* if it takes $[n]$-permutations to $[n]$-permutations. It is *non-decreasing* if $A \leq B \Rightarrow f(A) \leq f(B)$ where $A \leq B$ means that $A_{ij} \leq B_{ij}$ for all i and j.

Definition 5. Let $I \subseteq \{2, \ldots, k\}$ be an index set, and let $(f_i)_{i \in I}$ be non-decreasing permutation preserving linear maps from $\mathcal{M}_n$ to itself. A k-circular exchange economy is (I, f_i)*-restricted* if it is Σ-restricted where Σ is the set of k-circular permutation matrices σ of size $nk \times nk$ with the property that $\sigma'_{i,i\oplus 1} = f_i(\sigma'_{1,2})$ for all $i \in I$.

Theorem 5. *Every* (I, f_i)-restricted *k-circular exchange economy is balanced.*

Proof. Let $\mathcal{S}$ be the set of k-circular rational $nk \times nk$ matrices A such that $A'_{i,i\oplus 1} = f_i(A'_{1,2})$ for all $i \in I$. So Σ is the set of permutation matrices in $\mathcal{S}$. We will show that Σ is balanced. Then, by Theorem 4, it follows that the economy is balanced. Let A be any doubly stochastic matrix in $\langle \Sigma \rangle$. We must show that $A \in \langle \Sigma^{[nk]} \rangle$. If d is the least common denominator of the entries in A, then dA is a non-negative integer matrix whose rows and columns sum to d. If $d = 1$ we are finished so we can argue by induction on d, that is, we suppose that any doubly stochastic matrix in $\langle \Sigma \rangle$ whose entries have a common denominator smaller than d also belongs to $\langle \Sigma^{[nk]} \rangle$.

By the Birkhoff-von Neumann theorem, A can be written as a convex combination of $[nk]$-permutation matrices. Let π be any of these permutation matrices. Then $\pi \leq dA$ and π satisfies the first condition for lying in $\Sigma^{[nk]}$, namely $\pi'_{i,j} = \mathbf{0}$ unless $j = i \oplus 1$. To satisfy the second condition we have to play with π. Let σ be the $[nk]$-permutation matrix defined by

$$\sigma'_{i,i\oplus 1} = \begin{cases} f_i(\pi'_{1,2}) & \text{if } i \in I, \\ \pi'_{i,i\oplus 1} & \text{if } i \notin I. \end{cases}$$

By construction, $\sigma \in \Sigma^{[nk]}$. Since the f_i are linear maps, $\langle\Sigma\rangle \subseteq \mathcal{S}$, so $dA \in \mathcal{S}$. Since the f_i are non-decreasing, $\pi'_{1,2} \leq (dA)'_{1,2}$ implies, for every $i \in I$, $\sigma'_{i,i\oplus 1} = f_i(\pi'_{1,2}) \leq f_i((dA)'_{1,2}) = (dA)'_{i,i\oplus 1}$, which shows that $\sigma \leq dA$. The matrix $(dA - \sigma)/(d-1)$ is thus doubly stochastic and belongs to $\langle\Sigma\rangle$. The least common denominator of its entries is less than d. Hence, by the induction hypothesis, this matrix belongs to $\langle\Sigma^{[nk]}\rangle$, and so does $A = \frac{1}{d}((dA - \sigma) + \sigma)$. □

As a corollary we obtain our main result:

Theorem 6. *Every* (I, f_i)-restricted k*-circular exchange economy has a nonempty core.*

2.3. *Discussion*

For $k = 1$ and $I = \emptyset$, there are no restrictions at all and Theorem 6 simply restates Quinzii's Theorem 1 on ordinary exchange economies.

For $k = 2$, $I = \{2\}$, and $f_2(A) = A^{\mathrm{T}}$, the restriction says that if the ith agent in G_1 gets her house from the jth agent in G_2, then the jth agent in G_2 must get her house from the ith agent in G_1. In other words, houses are switched in pairs. In this case Theorem 6 transforms into Quinzii's Theorem 2 on the "pairing model". As Quinzii remarks, this is another form of the result of Kaneko (1982) that the "central assignment game" has a nonempty core.

For an original example, let $k = 2$ and $I = \{2\}$ as before. Let α and β be $[n]$-permutation matrices and put $f_2(A) = \beta A \alpha^{\mathrm{T}}$. This can be thought of as a strange market of some exotic commodity, which we call 'slithy toves' (from Lewis Carroll's poem *Twas Brillig*). In this market, the agents are n married couples such that every man has a best friend among the men (given by permutation α) and every woman has a best friend among the women (given by β), and for some reason the following rules apply: Men can buy slithy toves only from women, and vice versa, and the men adhere to the principle: "I buy your slithy tove only if my best friend's wife buys your best friend's husband's slithy tove". By Theorem 6 the slithy tove market has a nonempty core.

3. Conclusions

Rent control may affect people's market behavior in strange ways, as we discussed in Sec. 1.1. We have shown that, although Quinzii's general framework of house exchange economies does not include the exotic utility functions one can observe in the rent controlled Swedish housing market, her theory can be extended to prove nonemptiness of the core also in this real-world case. We then showed that the second theorem of Quinzii, on core nonemptiness of pairing models, can be generalized to include both her theorems (as well as other less realistic models such as our 'slithy tove' market) in one single result. As opposed to Quint's even more general characterization of exchange economies with nonempty core, this result is more specific and does not refer to balancedness in its formulation but is more in the spirit of Quinzii's original theorems.

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References

Bentzel, R., Lindbeck, A. and Ståhl, I. [1963] Bostadsbristen — en studie av prisbildningen på bostadsmarknaden, Almqvist & Wiksell, Stockholm, in Swedish.

Ellingsen, T. [2003] "Comment on Franz Hubert: Rent Control: Academic analysis and public sentiment," *Swedish Economic Policy Review* **10**, 83–86.

Ellingsen, T. and Englund, P. (eds.) [2003] *Swedish Economic Policy Review* **10**(1), Spring.

Eriksson, K. and Karlander, J. [2000] "Stable matching in a common generalization of the marriage and assignment models," *Discrete Math.* **217**, 135–156.

Eriksson, K. and Lind, H. [2004] Attityder till svarthandel med hyresrätter i Stockholm, preprint, in Swedish.

Gale, D. and Shapley, L. [1962] "College admissions and the stability of marriage," *Amer. Math. Monthly* **69**, 9–15.

Gale, D. [1984] Equilibrium in a discrete exchange economy with money, *Int. J. Game Theory* **13**, 61–64.

Glaeser, E. L. [2003] Does rent control reduce segregation?, *Swedish Economic Policy Review* **10**, 179–202.

Glaeser, E. L. and Luttmer, E. F. P. [2003] "The misallocation of housing under rent control," *Amer. Econ. Rev.* **93**, 1027–1046.

Kaneko, M. [1982] "The central assignment game and the assignment markets," *Journal of Mathematical Economics* **10**, 205–232.

Lind, H. and Hellström, A. [2003] Market rents and economic segregation: Evidence from a natural experiment, Working Paper TRITA-BFE-WP40, Division of Building and Real Estate Economics, KTH, Stockholm, Sweden.

Quint, T. [1997] "Restricted houseswapping games," *J. Math. Econ.* **27**, 451–470.

Quinzii, M. [1984] "Core and competitive equilibria with indivisibilities," *Int. J. Game Theo.* **13**, 41–60.

Roth, A. E. and Sotomayor, M. A. O. [1990] *Two-sided matching* (Cambridge University Press).

Scarf, H. [1967] The core of an N person game, *Econometrica* **35**, 50–69.

Shapley, L. and Scarf, H. [1974] "On cores and indivisibility," *J. Math. Econ.* **1**, 23–37.

Tufvesson, H. and Ljungkvist, T. [2001] Svarthandeln med hyreskontrakt, Stockholms fastighetsägareförening, Stockholm, Sweden, in Swedish.

Tijs, S., T. Parthasarathy, Potters, J. A. M. and Rajendra Prasad, V. [1984] "Permutation games: Another class of totally balanced games," *OR Spektrum* **6**, 119–123.